1. Find the maximum and minimum values of $f(x, y) = 81x^2 + y^2$ subject to the constraint $4x^2 + y^2 = 9$.

**Solution:** Using the Lagrange Multiplier Method, the candidate points are $(-\frac{3}{2}, 0), \left(\frac{3}{2}, 0\right), \left(0, -\frac{3}{2}\right), \left(0, \frac{3}{2}\right)$, and their $f$-values are $\frac{729}{4}, \frac{729}{4}, 9, 9$, respectively. Therefore the maximum value is $\frac{729}{4} = 182.25$ and the minimum value is 9.

2. Find the maximum and minimum values of $f(x, y) = 8x^2 - 2y$ subject to the constraint $x^2 + y^2 = 1$.

**Solution:** Using the Lagrange Multiplier Method, the candidate points are $\left(-\frac{3\sqrt{7}}{8}, -\frac{1}{8}\right), \left(\frac{3\sqrt{7}}{8}, -\frac{1}{8}\right), (0, -1), (0, 1)$, and their $f$-values are $\frac{65}{8}, \frac{65}{8}, 2, -2$, respectively. Therefore the maximum value is $\frac{65}{8} = 8.125$ and the minimum value is $-2$. 